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A BIRTH-LIFE-DEATH MODEL FOR THE EVALUATION AND PLANNING OF A HEALTH SERVICES PROGRAM

(Item 6 of the Agenda)

CONTENTS

Introduction and Objectives of the Study

The decision-maker is faced with the problem of lack of resources and the need to assign them multiply. If the consequences, i.e. outcomes, that result from the various possible changes of the decision variables were known, the problem would resolve itself in terms of selecting those changes that effect the most desirable results in terms of cost and/or effectiveness criteria.

In the real world, most of the decisions confronting a planner involve courses of action whose outcomes are not deterministic. Rather, the behavior of the outcome is best anticipated probabilistically. This is particularly true of actions over time.

The decision-maker wishes to determine which changes of health status and population patterns are most likely to result in desired alterations of morbidity, mortality, and life span (as well as quality of life). Ultimately, he must relate these expected benefits to the cost incurred in making the changes, so that he has a means of evaluating programs.

The following measures of mortality, life span, and "quality of life" resulting from change of health status and/or population pattern are considered in this paper:

- The age-stratified distribution of the population over time;
- The behavior over time of the number of deaths, by age groups and by causes, with emphasis on the change in the percentage composition;
- The behavior of specific mortality rates by age groups and by causes of death; and
- The fertility rates and population growth rates over time.

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* Paper prepared for the Ninth Meeting of the PAHO Advisory Committe on Medical Research by Jorge Ortiz, PAHO Department of Research Development and Coordination, and Rodger D. Parker, Johns Hopkins University School of Hygiene and Public Health.

The first phase of the PAHO cost-effectiveness study reported here has the following purposes:

- To develop a comprehensive mathematical model in which changes of decision (or control) variables representing health status and/or population programs are related to changes in the measures of mortality, life span, and quality of life. Such a model involves a Markovian representation of the birth-life-death process; and
- To develop a simple mathematical model relating the impact on life expectancy due to changes in decision variables.

The System

A system is defined as "a set of intercommunicating states that constitute a whole."

The birth-life-death process is a system in which the states are the age intervals of life and death by causes of disease. The intercommunication between the age interval states and the death states is represented in the form of probabilities of death for each cause of death within each age interval. These probabilities determine the over-all survival rate to the next age interval state.

The age intervals correspond to age groups as follows:

Interval	Age group
1	4 and under
2	5-9
3	10-14
.	.
.	.
.	.
17	80-84
18	85 or above

In line with customary PAHO practice, the causes of death are classified into the following groups:

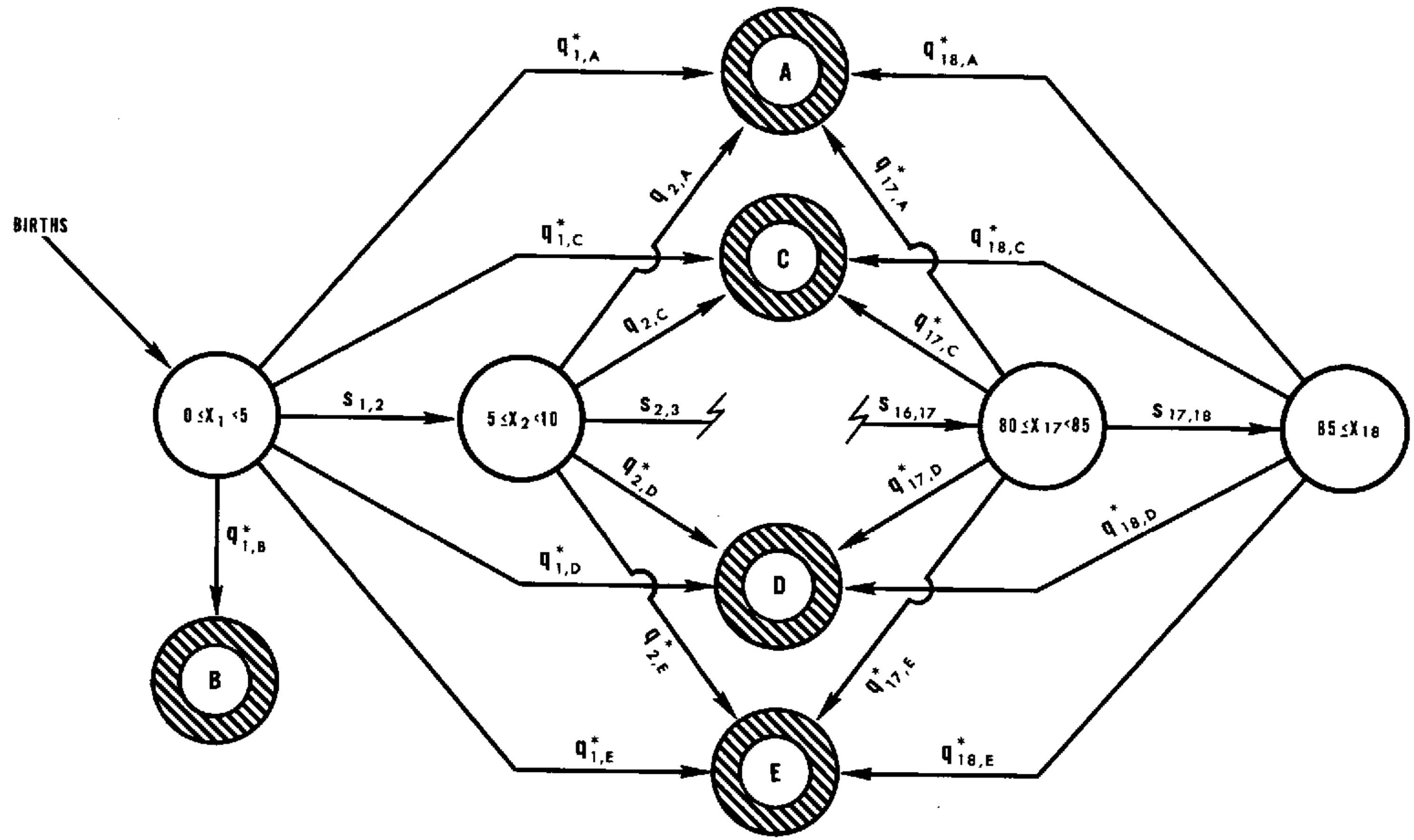
A = Infectious and parasitic diseases, pneumonia, influenza, bronchitis, and gastroenteritis

B = Diseases of early infancy

C = Tumors

FIGURE 1

GRAPHIC REPRESENTATION OF THE SYSTEM BIRTH-LIFE-DEATH



Xi= REPRESENTS STATE (i= 1, 2,... 18) OF AGE INTERVALS
A= STATE OF DEATHS BY INFECTIOUS AND PARASITIC DISEASES
B= STATE OF DEATHS BY DISEASES OF THE EARLY INFANCY
C= STATE OF DEATHS BY TUMORS
D= STATE OF DEATHS BY CARDIOVASCULAR DISEASES
E= STATE OF DEATHS BY ALL OTHER DISEASES

D = Cardiovascular diseases

E = All others

The system is depicted in Figure 1. In this diagram the age states are represented by unshaded circles, and the categories of death by shaded circles with the corresponding letter of identification. The variable X_i stands for the ages within the age interval. The intercommunication among categories is shown by arrows. Over the arrows two types of symbols are found:

q^*_{ij} = The estimate of the probability of death due to cause of death j, in age interval i ($i=1,\ldots,w,\quad j=A,B,C,D,E$)

and

$S_{i,i+1}$ = The estimate of the probability of surviving from age interval i to age interval i +1

where w is the number of intervals of age.

Of course, any individual, at any given time, may be in only one state. A system can be either open or closed. A closed system is one without any inputs, i.e. births or immigrations.

Thus the average time experience (or life table) of a single entry (or cohort) may be traced. The model generates the probability distribution of death due to each cause within each age category, and the life expectancy. An open system has both inputs and outputs. The present study embodies an open system, as shown in Figure 1. Inputs in the form of births and outputs in the form of deaths, due to the five groups of diseases, are included; however, immigration and migration are not considered. States with arrows pointing both in and out are nonabsorbing states in Markov chain terminology. Those with arrows pointing only in (the death states) are called absorbing states.

In finding a solution for the system, one desires knowledge of the dynamic behavior of the numbers and percentages of deaths by causes of disease of the people in the various age interval states, as well as elucidation of the mortality, birth, and growth rates. It is convenient to distinguish between transient and steady state properties of the solution. A steady state property is one that essentially holds for all time after a certain point in the future. For our purposes, the steady state situation of interest is one in which the percentage composition of the population by age intervals is a constant. The transient situation is that period in time before the steady situation is entered.

Two sets of decision variables control our system:

- Specific mortality rates by age intervals and groups of diseases (abbreviated hereafter as SMRAC)
- Specific fertility rates by age interval (hereafter SFRA)

For any change in the set of decision variables representing a projected change in the health services system, the vital statistics in the steady state situation are particularly useful measures for characterizing the impact, since they predict the more of less permanent effect of the change. For example, the impact of changing specific fertility rates alone will alter the age distribution of the population, which in turn will alter the total death rates for each disease grouping; however, a steady state total death rate for each group will be approached as time goes on. This figure will be one of importance in planning the needed new program effort in each disease group.

Changes affecting the health status of the population imply modification of the decision variables (SMRAG's and SFRA's). On the other hand, a change in the level of the SMRAG's and/or SFRA's may be viewed as an experiment in which the outcome is measured in terms of the impact on one or more of the population structures, mortality structures, or quality of life (chosen as criteria of impact for the experiment). The experiment is simulated by applying the birth-life-death process model derived in this study. Thus an experimental design program for obtaining optimal conditions using any given criteria can be carried out via simulation.

The First Model

The mathematical model considered adequate to analyze mortality structures by groups of causes of death and other vital statistics within each time interval, for an arbitrary length of time into the future, incorporates a finite Markov chain with absorbing barriers (the five states of death) and a forcing function representing births.

Table 1-A

TRANSITION PROBABILITY MATRIX

Stage t + 1

	State A	State B		State E	State 1st	State 2nd	State 3rd		State 17th	State 18th
State A	1									
State B		1								
.		.								
.			.							
.			.							
State E				1						
State 1	$q^*_{1,A}$	$q^*_{1,B}$	. . .	$q^*_{1,E}$	$b_{1,1}$	$S_{1,2}$	0	. . .	0	0
State 2	$q^*_{2,A}$	$q^*_{2,B}$	. . .	$q^*_{2,E}$	$b_{2,1}$	0	$S_{2,3}$	. . .	0	0
State 3	$q^*_{3,A}$	$q^*_{3,B}$	. . .	$q^*_{3,E}$	$b_{3,1}$	0	0	. . .	0	0
.	.	.		.	.	.	.			
.	.	.		.	.	.	.			
.	.	.		.	.	.	.			
State 17	$q^*_{17,A}$	$q^*_{17,B}$	. . .	$q^*_{17,E}$	$b_{17,1}$	0	0		$S_{17,18}$	0
State 18	$q^*_{18,A}$	$q^*_{18,E}$	. . .	$q^*_{18,E}$	0	0	0	. . .	0	0

Table 1-B

INITIAL STATE VECTOR

$D_{0,A}$	Number of deaths in Group A
$D_{0,B}$	Number of deaths in Group B
$D_{0,C}$	Number of deaths in Group C
$D_{0,D}$	Number of deaths in Group D
$D_{0,E}$	Number of deaths in Group E
$W_{0,1}$	Population in State 1
$W_{0,2}$	Population in State 2
.	
.	
.	
$W_{0,18}$	Population in State 18

The salient characteristics of the model are (1) that it represents a dynamic process in time in which transitions between age brackets, and to various types of death, occur probabilistically; and (2) that the outcome at each stage in time depends only on the outcome at the previous stage in time and on the transition probabilities plus birth rates.

The model is the analytical representation of the system depicted in Figure 1. The number of entries into each circle at each stage in time is determined by applying matrix C, given in Table 1-A, to the state vector, whose components are the numbers of people in each circle during the previous stage in time.

The initial stage, represented by numbers of people in each circle at the initial time for the system, is given in Table 1-B. Π_o denotes the state vector at this time (t = o) and Π_t denotes the state vector at any time (t) thereafter.

Matrix C in Table 1-A has an upper left-hand submatrix symbolized by I. The five rows of this submatrix have all zero elements except a 1 in the column entry having the same letter as the row. This simply means that the probability of remaining dead is one, or certain--i.e. a person dead at any given stage remains dead throughout all future stages. The submatrix directly to the right of this submatrix contains all zeroes and is symbolized by O.

The rows beneath the first five may also be thought of as being split into two submatrices, one to the left and one to the right. The left submatrix contains the elements (q^*_{ij}), which are the probabilities of death due to a group of causes (j) for members of a given age bracket (i) during a fundamental time period of length equal to the length of the age brackets--in these examples, five years. This submatrix, denoted by Q, is the absorption matrix. The entries of this submatrix are estimated by formulas involving the specific mortality rates (see Appendix I) by age interval and group of causes of death.

The lower right-hand submatrix has elements $S_{i,i+1}$ directly above the diagonal, which are the probabilities of an individual in each bracket (i) surviving a fundamental time period and thus entering the next age bracket (i + 1). If all other elements of this lower right-hand submatrix are zero, then it is called the surviving submatrix S, and the submatrix is used in this form for the closed system. For an open system, this right-hand submatrix will also contain in the first column elements denoted by $b_{i,1}$, which are functions of the age--specific fertilities and survival probabilities (see Appendix II). These elements generate births.

Matrix S is useful in conjunction with matrix Q for estimating the fraction of people in each age bracket (i) who will ultimately die due to each group of causes of death (j). (These fractions are the elements of the matrix $(I-S)^{-1} Q$.) See Figure 26.

Table 1-B exhibits the components of the initial state vector π_o, ($\pi_o = D_{OA}, D_{OB}, \ldots, D_{OE}, W_{O1}, W_{O2}, \ldots, W_{O18}$). In general, D_{tj} is the cumulative number of deaths due to a given group of causes of death (j) at the t^{th} stage in time. Thus, we take $D_{Oj} = 0$, $j = A, B, \ldots, E$. Also, W_{ti} is the number of women in a given age bracket (i) at the t^{th} stage in time, so that W_{Oi}, $i = 1, 18$ gives the initial population of women.

A simulated experiment is then set up by specifying the SMRAG's, which determine the Q matrix and the S matrix, and the fertility functions b_{i1}, which include the SFRA's. The behavior of the population at a given stage (t) is determined in terms of the state vector

$$(\pi_t = D_{tA}, D_{tB}, \ldots, D_{tE}, W_{t1}, W_{t2}, \ldots, W_{t18})$$

which is calculated recursively by multiplying the row vector π_{t-1} of the previous stage by the matrix C. The number of deaths (d_{tj}) due to a given cause (j) occurring during the time interval between t - 1 and t is expressed as

$$d_{tj} = D_{tj} - D_{t-1,j}$$

since D_{tj} is the cumulative total of deaths due to cause j that took place at time t.

The rest of the vital statistics parameters of interest (fertility rates, mortality rates, growth rates, percentage composition of deaths, etc.) are functions of the components of π_t, and their calculation is incorporated within the computer program.

The solution for Π_t is symbolically stated as

$$\Pi_t = \Pi_{t-1} C$$

Alternately, because C is a constant matrix,

$$\Pi_t = \Pi_o C^t$$

Thus, the entire behavior of the birth-life-death process depends only on the initial situation expressed by Π_o and the powers of matrix C.

The Second Model

Life expectancy, needless to say, is one of the important measures of human well-being. One of the fundamental goals set forth in the Charter of Punta del Este is to increase life expectancy at birth throughout Latin America by a minimum of five years within the first decade of the Alliance.

Life expectancy (e_i) represents the average remaining lifetime in years of a person who survives to the beginning of the i^{th} age interval. A general expression for e_i is derived in Appendix III, where more detailed definitions are given.

Life expectancy at birth is e_1 in the present notation, and it is given by

$$e_1 = n_1 P_1^* + n_2 P_2^* P_{1,2} + n_3 P_3^* P_{1,2} P_{2,3} + \cdots + n_{\omega-1} P_{\omega-1}^* P_{1,2} P_{2,3} \cdots P_{\omega-1,\omega} + P_{1,2} P_{2,3} \cdots P_{\omega-1,\omega} \left(1/M_{\omega.} \right)$$

or, more completely,

$$e_1 = \sum_{i=1}^{\omega-1} n_i P_i^* \prod_{r=0}^{i-1} P_{r,r+1} + \prod_{r=0}^{\omega-1} P_{r,r+1} \left(1/M_{\omega.} \right)$$

where

n_i = size of the i^{th} age interval (all five years except the last),

P_i^* = $(P_{i,i+1} + a_i q_{i,.})$ which is a function of the transition probabilities

In Costa Rica (1963), 34 per cent of all female deaths corresponded to deaths from infectious and parasitic diseases among children under 5. A further examination of deaths due to Group A causes in the first age bracket shows that 78 per cent of the deaths are attributed to categories B36, B31, and B17 of the International Abridged List 1955 of the International Classification of Diseases. The breakdown is as follows:

DEATHS WITHIN THE FIRST AGE INTERVAL (UNDER 5 YEARS)
DUE TO CAUSES IN GROUP A

	Subset of diseases of Group A	Percentage contribution to total deaths in Group A
B36	Gastroenteritis, duodenitis, enteritis, colitis (except diarrhea of newborn)	45.8
B31	Pneumonia	19.4
B17	All infectious and parasitic diseases not contained in other subgroups of Group A	13.3
All other	B11, to B16, B30, B32	21.5
		100

It can be seen that a substantial reduction in specific mortalities within the first age interval from causes in categories B36, B31, and B17 would have a profound impact on the national health status of Costa Rica.

The next step, then, is to postulate a Level II of the SMRAG's that will involve altering these specific mortalities to a level corresponding more or less to that suggested by 1966 values in Puerto Rico for age interval 1 but somewhat higher. This is done by taking as the new rate for B36, 30 per cent of the old rate; as the new rate for B31, 30 per cent; and as the new rate for B17, 51 per cent. The impact of this reduction is to lower the specific mortality rate for Group A for females under 5 years of age from the baseline value of 1,450 per 100,000 female population to 695 per 100,000. This is a reduction of 52 per cent.

Table 3-A

TRANSITION MATRIX FOR BASELINE DATA
(all entries have been multiplied by 1,000)

Stage t	State A	State B	State C	State D	State E	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8	State 9	State 10	State 11	State 12	State 13	State 14	State 15	State 16	State 17	State 18
						Stage t + 1																	
State A	1																						
State B		1																					
State C			1																				
State D				1																			
State E					1																		
State 1	14.190	4.457	0.095	0.135	2.570	0	978.551																
State 2	2.531	0	0.217	0.180	1.591	0.887		995.479															
State 3	1.226	0	0.340	0.408	1.704	139.018			996.319														
State 4	0.933	0	0.254	0.593	3.307	529.679				994.911													
State 5	0.716	0	0.238	1.194	5.256	811.426					992.594												
State 6	1.741	0	0.995	1.741	5.349	762.214						990.172											
State 7	1.255	0	2.651	1.395	6.140	603.191							988.557										
State 8	3.128	0	3.128	2.606	6.429	383.312								984.707									
State 9	2.684	0	6.935	4.027	8.501	142.110									977.851								
State 10	4.346	0	12.124	4.117	6.405	19.440										973.006							
State 11	3.766	0	14.751	9.415	10.357	0											961.708						
State 12	7.932	0	21.319	18.840	19.831	0												932.076					
State 13	10.527	0	29.389	38.161	23.248	0													898.674				
State 14	16.410	0	52.514	50.052	45.129	0														835.893			
State 15	30.832	0	54.363	88.441	50.306	0															776.055		
State 16	45.944	0	65.971	104.847	81.285	0																701.952	
State 17	126.669	0	68.401	260.938	131.736	0																	412.257
State 18	260.586	0	120.521	338.762	280.130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 3-B - INITIAL-STATE VECTOR

π_0 = [0; 0; 0; 0; 0; 122,889; 107,872; 85,052; 66,435; 53,158; 43,604; 38,688; 34,430; 26,792; 22,530; 20,600; 13,345; 12,721; 7,882; 5,843; 4,685; 2,377; 864]

An experimental Level II of the SFRA was elicited on the basis of figures from Chile for 1964. Chile was selected because it is a Latin American country which is not too dissimilar from Costa Rica in terms of ecological make-up but which, at the same time, has more idealized birth rates. It is rational to hypothesize that a similar birth structure would be feasible in Costa Rica. The 1963 baseline data for Costa Rica in terms of the transition matrix and the initial state vector are given in Table 3 (A and B).

The two levels of SFRA's are as follows:

		Age interval							
		10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
S F R A	Costa Rica, 1963 (Level I)	.00080	.12435	.35253	.37806	.30862	.23508	.11040	.01744
	Costa Rica, 1963 (Level II)*	.00068	.10707	.2553	.19192	.11332	.817	.03196	.0064

*Corresponding to the Chilean SFRA values for 1964

Discussion: Results in the Transient Situation

A list of figures, defined by their abscissas (x-axes) and ordinates (y-axes), is given in Exhibit II for ready reference to the experimental results. Generally, a planner when he is designing an experiment will have in mind a number of effectiveness criteria that he wishes to meet by a given health services action. These criteria may not always be consistent: for example, there may be a desire to simultaneously lower a death rate and to reduce a growth rate. Usually the results of the experiment will be mainly quantitative, i.e. primarily useful in giving numerical estimates of mortality and other demographic variables that otherwise might only be guessed at. Concomitantly, in some instances inferences of a qualitative type may be sought. Typical criteria might include the following:

(a) Diminution of the total death rate
(b) Diminution of the death rate due to one of the groups of causes
(c) Alteration of the age composition of deaths
(d) Increase of life expectancy
(e) Decrease of total population
(f) Increase of total population
(g) Alteration of the over-all age distribution

In Exhibit I some of the figures that would be useful in demonstrating the impact of the programs implied by our hypothetical experiment on each of the above criteria are given.

Exhibit I

Criterion	Figure
(a)	17
(b)	18, 19, 20, 21, 22
(c)	12, 13, 14, 15, 16
(d)	2, 3
(e)	3
(f)	3
(g)	4, 5

As an example of a logical inference--as opposed to a numerical quantification--that might be sought in running an experiment, the administrator of a program on cancer might take as his criterion the lowering of the death rate due to tumors in his country. He knows that there are four basic qualitative possibilities within his country for the future: age-specific mortality due to malignant tumors may remain more or less the same or it may go down, and specific fertility may remain the same or it may go down. Loosely speaking, this corresponds to a situation analogous to that in the experimental design:

Situation	Specific mortality (cancer)	SFRA
1	Same	Same
2	Same	Down
3	Down	Down
4	Down	Same

Since lower specific fertility would logically imply an ultimately older population, which in turn means a higher death rate due to tumors, and since lower specific mortality would imply a reduced death rate, he can logically infer the following steady state possibilities within his country:

Situation	Death rate (cancer)
2	Highest
1 and 3	Between 2 and 4
4	Lowest

What he cannot infer is whether or not the death rate for cancer will be higher in Situation 1 or in Situation 3. This logical question is answered, along with numerical specifications, if the appropriate simulations are made.

Because of limitations in time available to prepare this report, and also partly because of its expository nature, a thorough discussion of the insights that could be inferred through careful examination of each of the figures has not been included. It has been decided to only list typical outcome observations in Exhibit II, by way of indicating the kind of discussion that might be developed at greater length.

EXHIBIT II

Figure	y-axis	x-axis	Experiments	Outcome
2	Life expectancy (years)	Age (years)	1,4	Experimental curve 4 shows 2.35 years' increase in life expectancy at birth over baseline.
3	Total population (100,000's)	Time (5-year intervals)	All	Distance between curve 4 and curve 1 shows lives saved, etc.; baseline population doubles every 17 years, Situation 2 population doubles every 28 years.
4	Percentage of population under 5 years of age	"	"	For baseline SFRA's percentage is virtually a constant; for reduced fertilities curve is unimodal.
5	Percentage of population over 65 years of age	"	"	
6	Total deaths (1,000's)	"	"	The sum of the difference of total deaths between curve 1 and curve 4 during the time period will give the total number of potential lives to be saved over that period by the application of the program associated with Situation 4.
7	Deaths due to infectious and parasitic diseases	"	"	Cross-over point between curve 2 and curve 4 is explained by the behavior of fertility rates in that period (see Figure 23). Because deaths occur basically in the young age intervals, Situation 4 with lower group A death rates but higher birth rates ultimately yields a younger population which has more Group A deaths than is the case in Situation 2.
8	Deaths due to diseases of the early infancy (1,000's)	"	"	

Figure	y-axis	x-axis	Experiments	Outcome
9	Deaths due to tumors (1,000's)	Time (5-year intervals)	All	
10	Deaths due to cardiovascular diseases (1,000's)	"	"	
11	Deaths due to all other causes (1,000's)	"	"	
12	Percentage of deaths due to infectious and parasitic diseases (1,000's)	"	"	
13	Percentage of deaths due to diseases of early infancy	"	"	
14	Percentage of deaths due to tumors	"	"	The eventual impact of decreased fertility rates is to increase the percentage of deaths due to cancer because of an aging population.
15	Percentage of deaths due to cardiovascular diseases	"	"	
16	Percentage of deaths due to all other causes	"	"	
17	Annual total mortality rate per 100,000	"	"	Experimental Situations 2 and 3 show an upward trend mainly due to increased cancer and cardiovascular death rates because of an increasingly aging population.
18	Annual mortality rate per 100,000 due to infectious and parasitic diseases	"	"	

Figure	y-axis	x-axis	Experiments	Outcome
19	Annual mortality rate per 100,000 due to diseases of early infancy	Time (5-year intervals)	All	
20	Annual mortality rate per 100,000 due to tumors	"	"	
21	Annual mortality rate per 100,000 due to cardiovascular diseases	"	"	
22	Annual mortality rate per 100,000 due to all other causes	"	"	
23	Annual fertility rate per 1,000	"	"	Curves 2 and 3 are unimodal with the maximum within the period 1978-1983; the curves are increasing up to this point, which is roughly 15 years after the advent of the program, at which time the diminished number of babies produced by lowered SFRA's has come to child-bearing age.
24	Annual growth rate per 1,000	"	"	
25	Probability of dying during the age interval in each of the five groups of diseases	"	"	These probabilities constitute the basic input for the estimation of life expectancy
26	Probability of eventually dying in each of the five groups of diseases	Age (years)	1	The sum of these probabilities for each age is one, and they yield an estimate of the percentage of people at each age who will die from each cause of death.

FIGURE 2

AVERAGE NUMBER OF YEARS OF FUTURE LIFE IN THE FEMALE POPULATION OF COSTA RICA

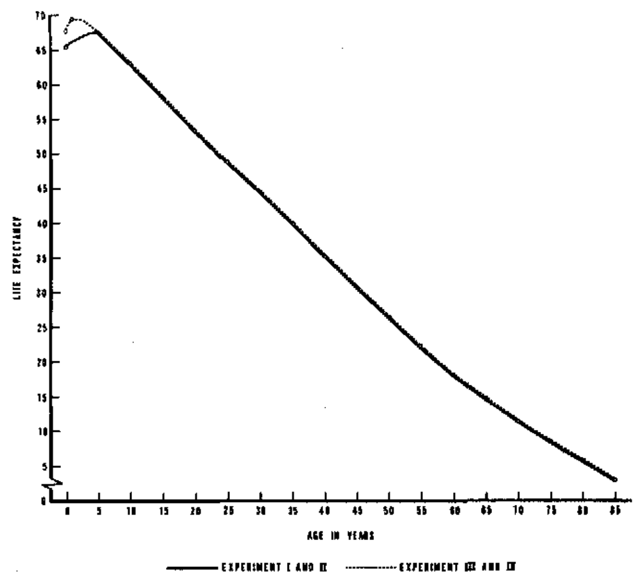


FIGURE 3

PROJECTIONS FOR THE FEMALE POPULATION OF COSTA RICA

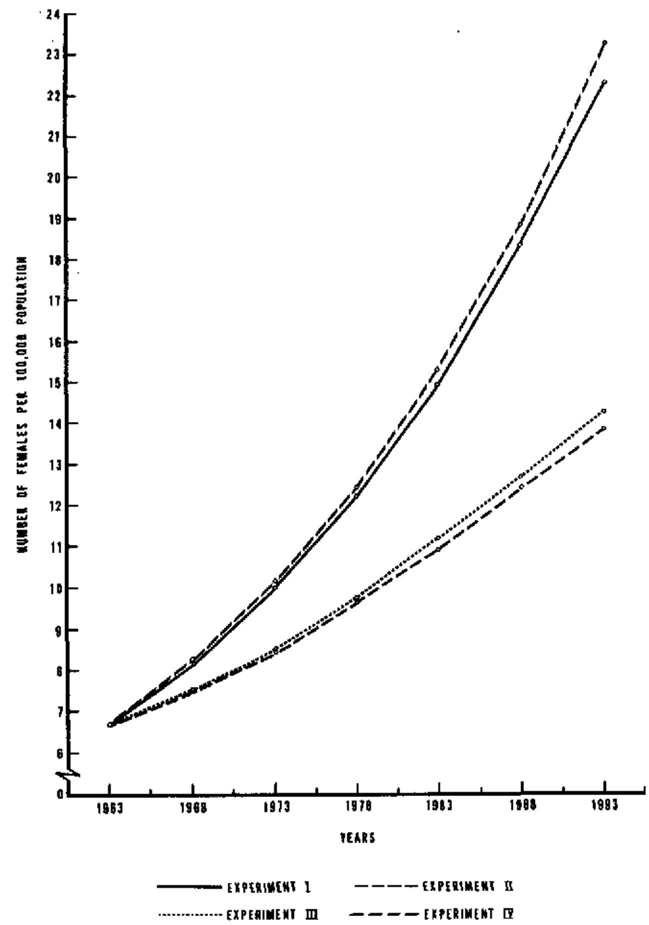


FIGURE 4

PROJECTIONS OF PERCENTAGES OF FEMALE POPULATION UNDER 5 YEARS OF AGE

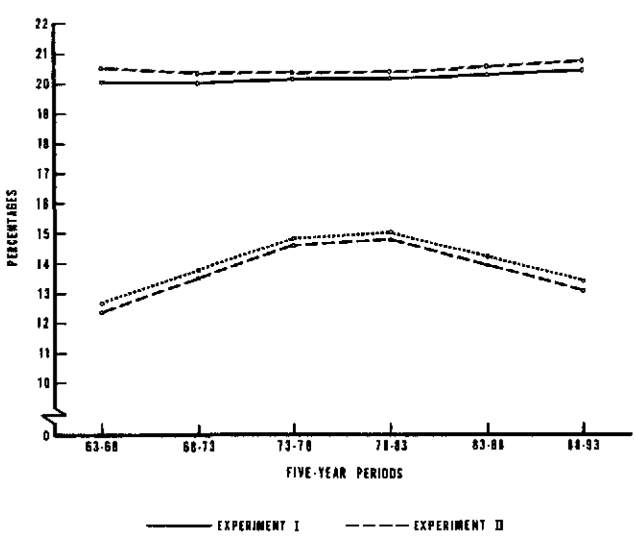


FIGURE 5

PROJECTIONS OF PERCENTAGES OF FEMALE POPULATION OVER 65 YEARS OF AGE

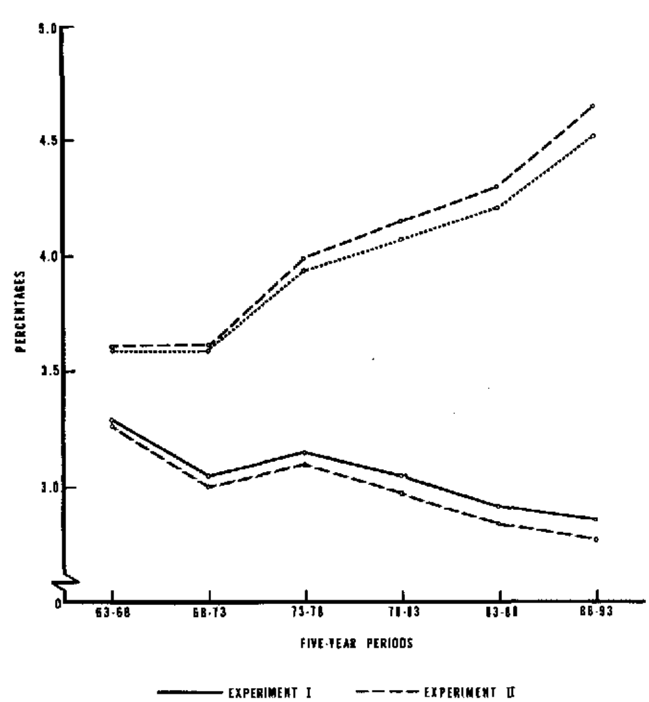
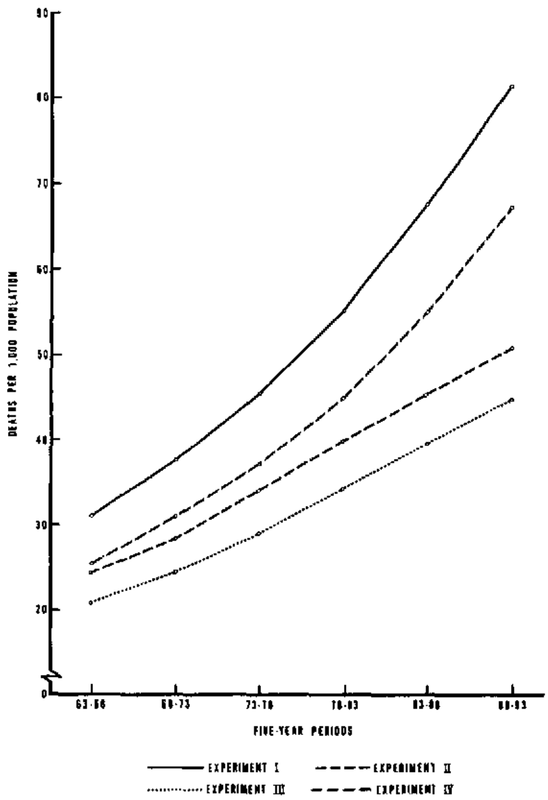


FIGURE 6

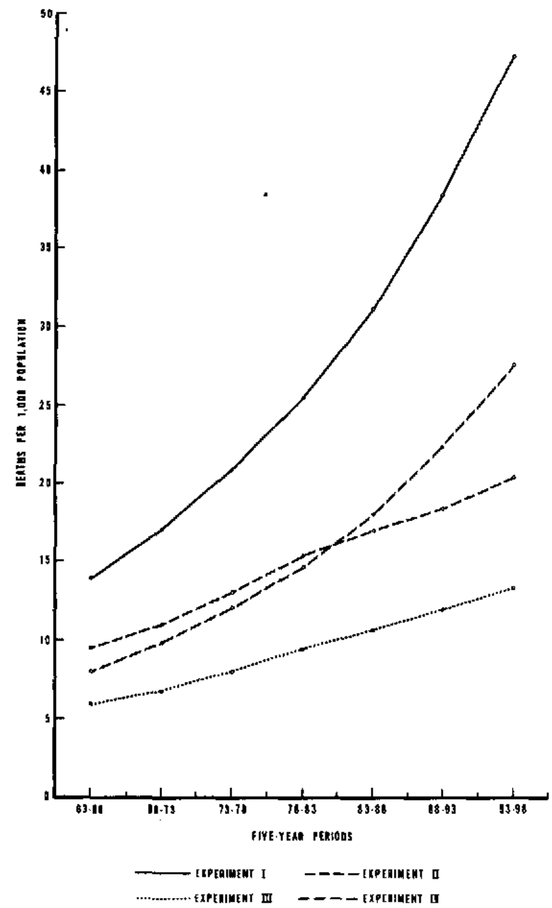
PROJECTIONS OF THE TOTAL NUMBER OF DEATHS BY FIVE-YEAR PERIODS*



*FEMALE POPULATION OF COSTA RICA

FIGURE 7

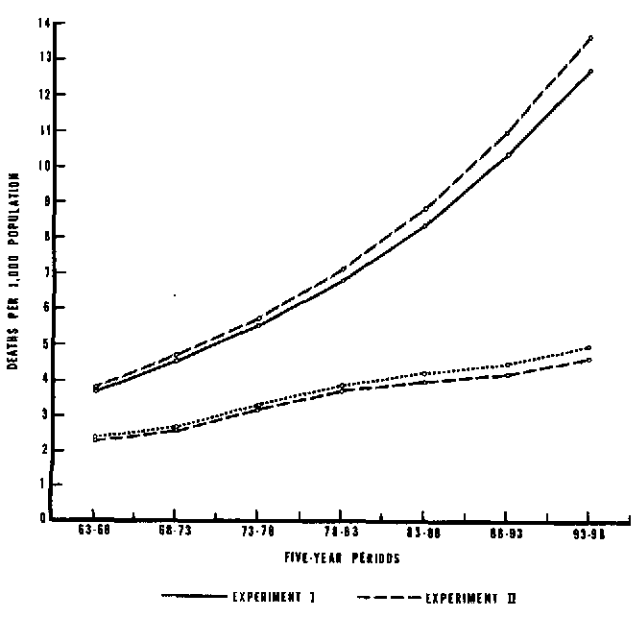
PROJECTIONS OF THE NUMBER OF DEATHS CAUSED BY INFECTIOUS AND PARASITIC DISEASES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 8

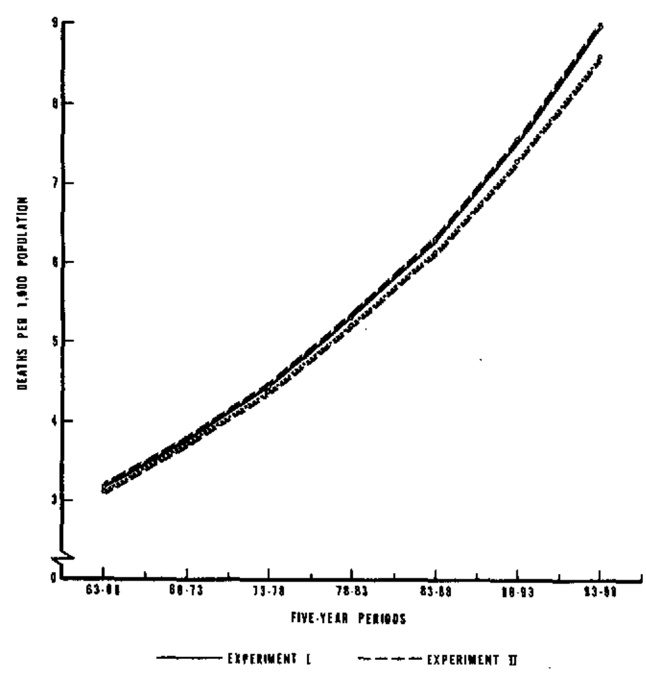
PROJECTIONS OF THE NUMBER OF DEATHS CAUSED BY DISEASES OF EARLY INFANCY BY FIVE-YEAR PERIODS*



*FEMALE POPULATION OF COSTA RICA

FIGURE 9

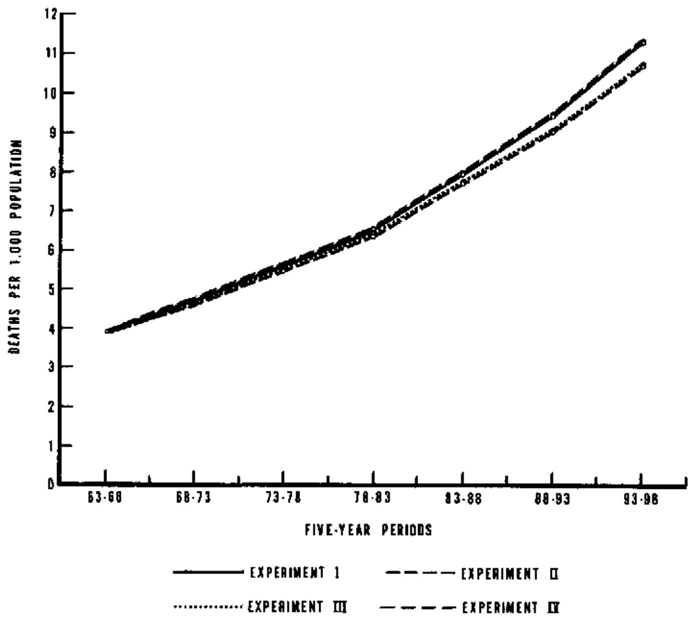
PROJECTIONS OF THE NUMBER OF DEATHS CAUSED BY TUMORS BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 10

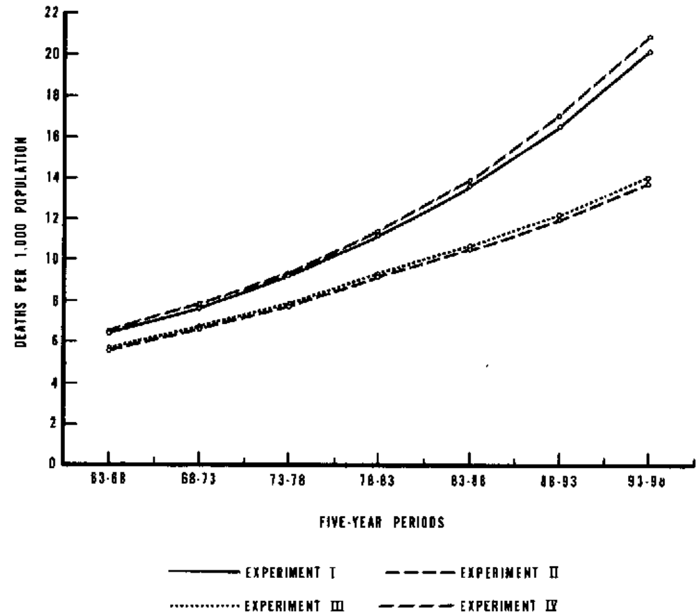
PROJECTIONS OF THE NUMBER OF DEATHS CAUSED BY CARDIOVASCULAR DISEASES BY FIVE-YEAR PERIODS*



*FEMALE POPULATION OF COSTA RICA

FIGURE 11

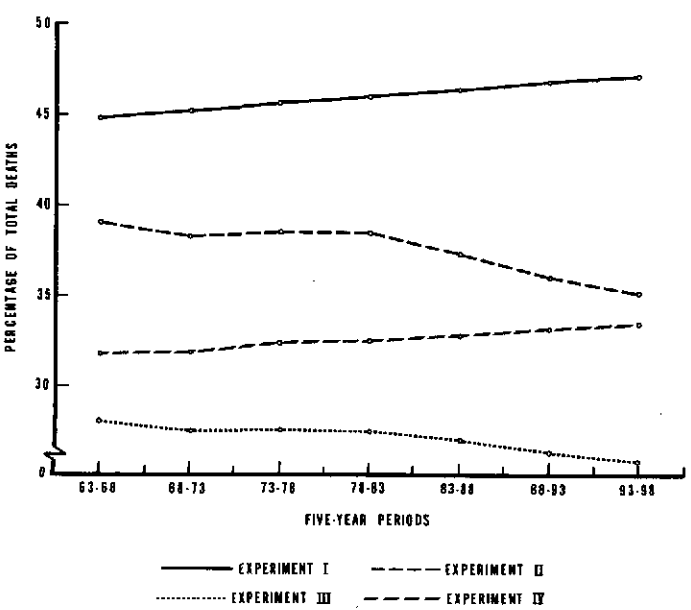
PROJECTIONS OF THE NUMBER OF DEATHS CAUSED BY ALL OTHER DISEASES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 12

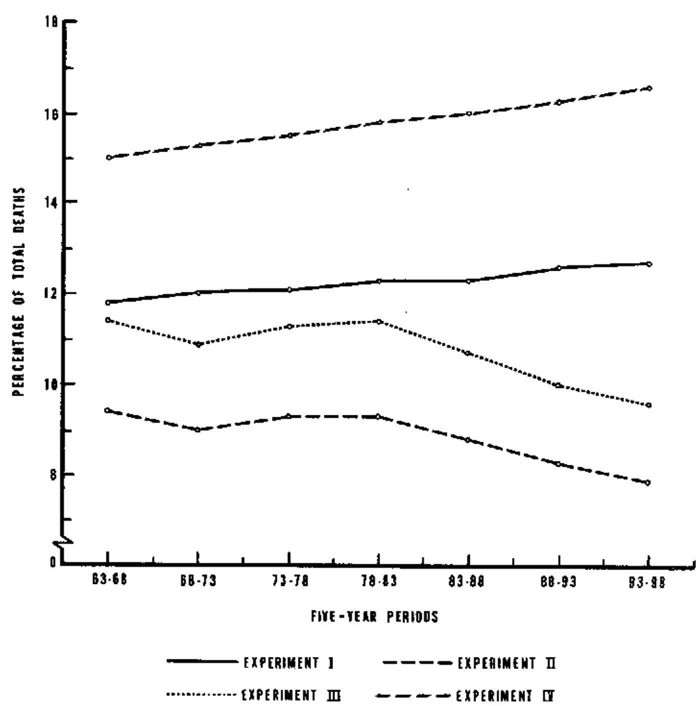
PROJECTIONS OF THE PERCENTAGE OF TOTAL DEATHS CAUSED BY INFECTIOUS AND PARASITIC DISEASES BY FIVE-YEAR PERIODS*



*FEMALE POPULATION OF COSTA RICA

FIGURE 13

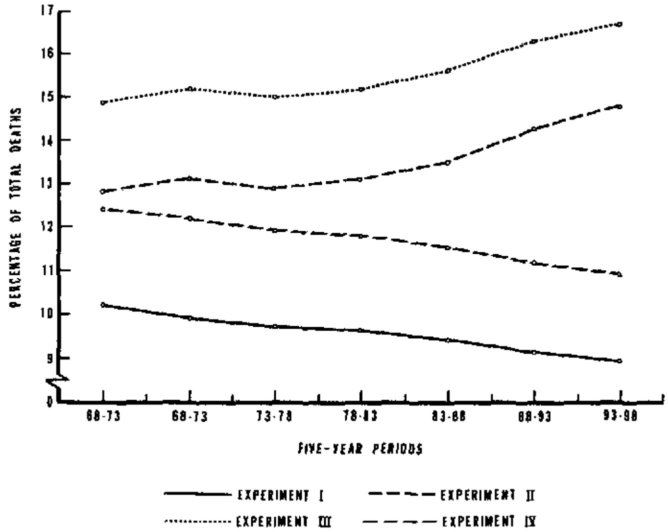
PROJECTIONS OF THE PERCENTAGE OF TOTAL DEATHS CAUSED BY DISEASES OF EARLY INFANCY BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 14

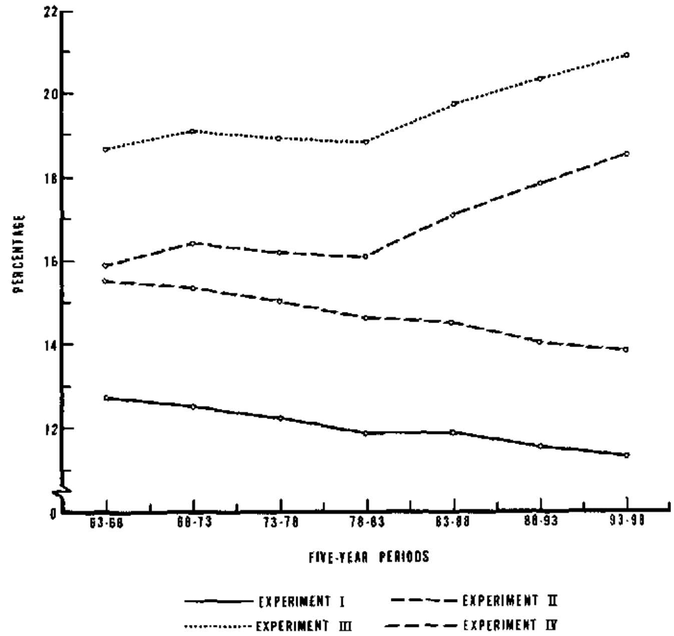
PROJECTIONS OF THE PERCENTAGE OF TOTAL DEATHS CAUSED BY TUMORS BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 15

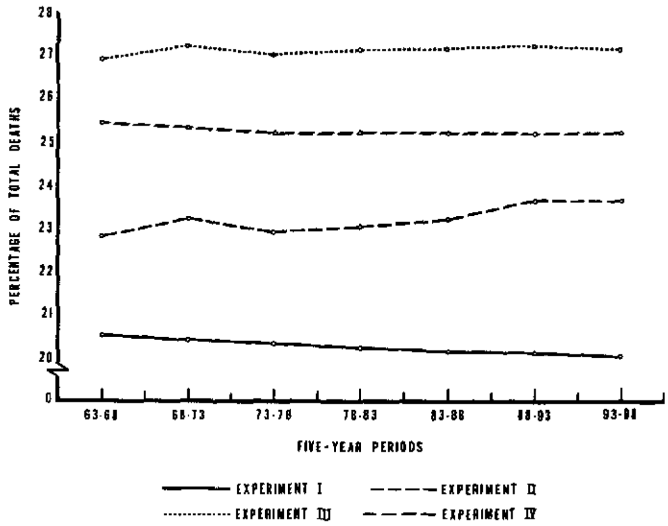
PROJECTIONS OF THE PERCENTAGE OF TOTAL DEATHS CAUSED BY CARDIOVASCULAR DISEASES BY FIVE-YEAR PERIODS*



*FEMALE POPULATION OF COSTA RICA

FIGURE 16

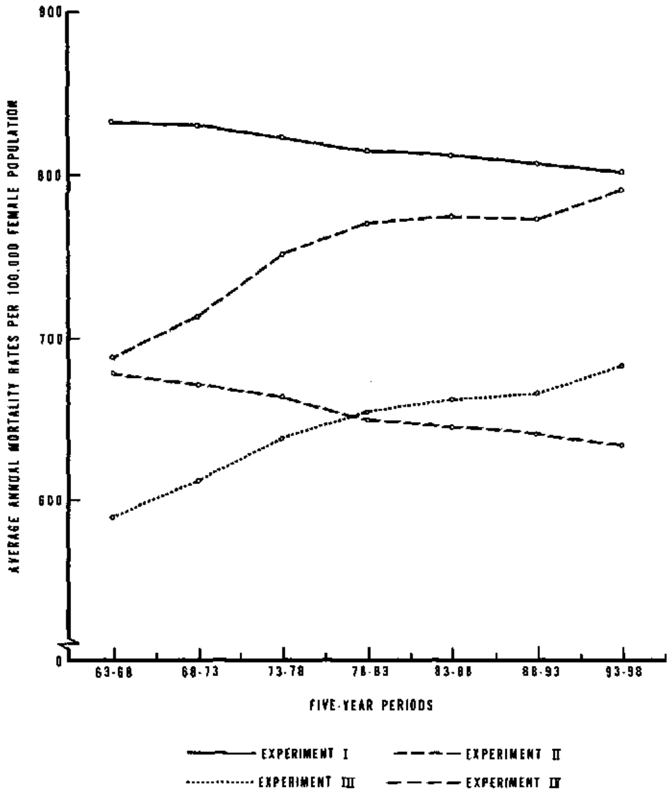
PROJECTIONS OF THE PERCENTAGE OF TOTAL DEATHS CAUSED BY ALL OTHER DISEASES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 17

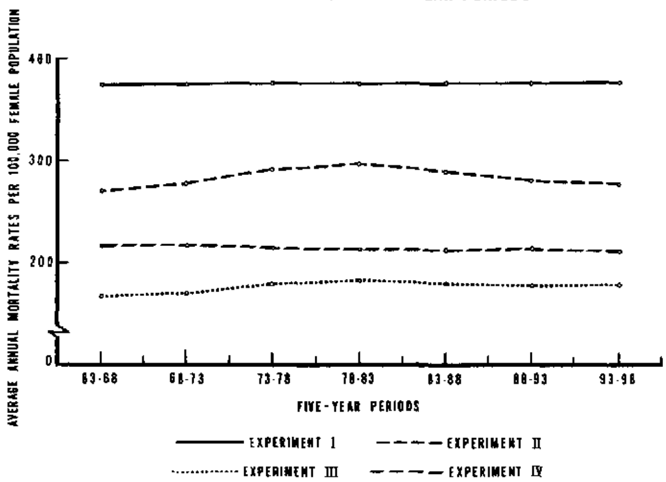
PROJECTIONS OF THE TOTAL ANNUAL AVERAGE MORTALITY RATES BY FIVE-YEAR PERIODS*



*FEMALE POPULATION OF COSTA RICA

FIGURE 18

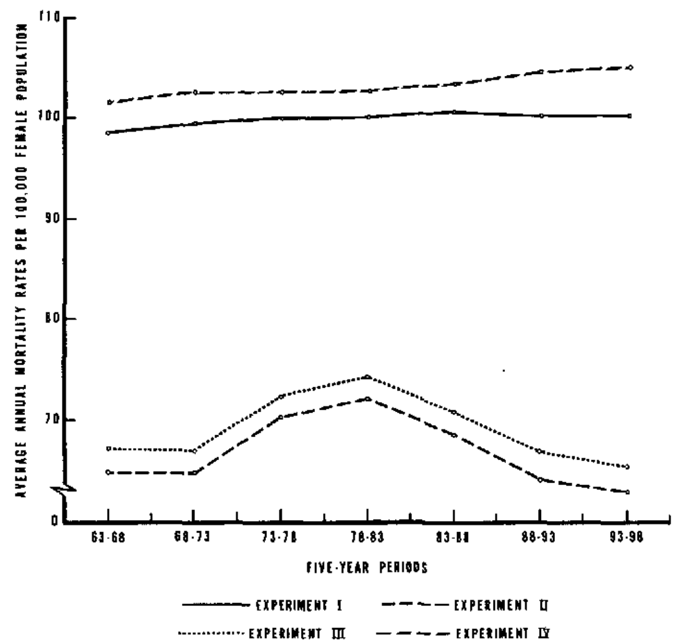
PROJECTIONS OF AVERAGE ANNUAL MORTALITY CAUSED BY INFECTIONS AND PARASITIC DISEASES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 19

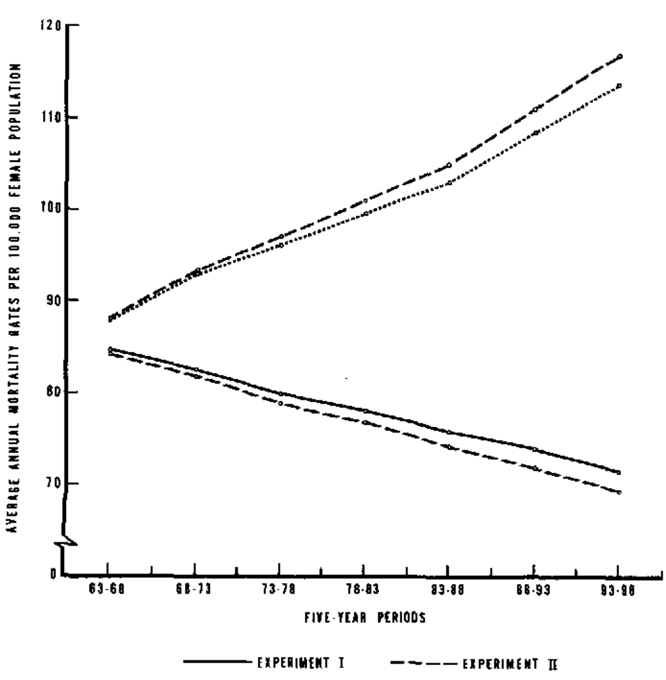
PROJECTIONS OF AVERAGE ANNUAL MORTALITY CAUSED BY DISEASES OF EARLY INFANCY BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 20

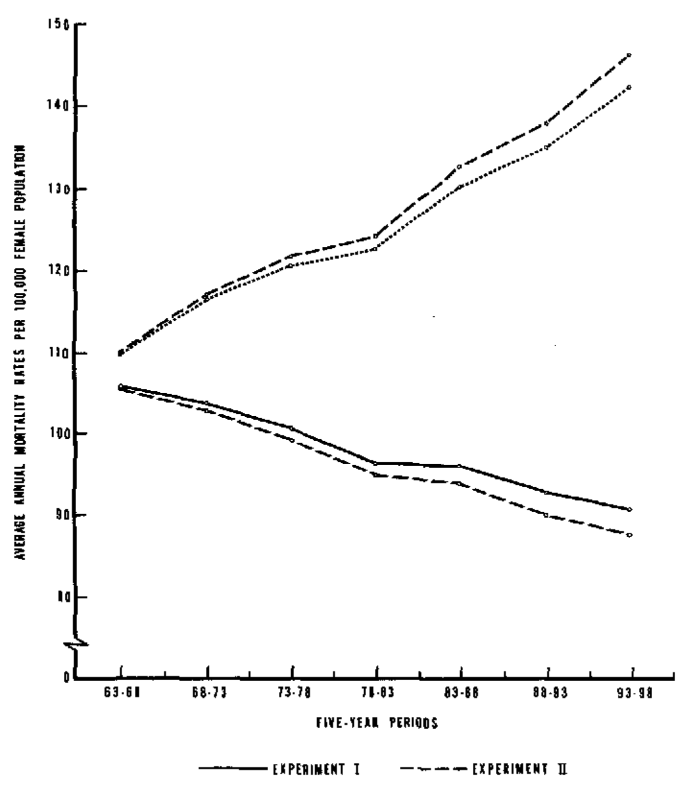
PROJECTIONS OF AVERAGE ANNUAL MORTALITY CAUSED BY TUMORS BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 21

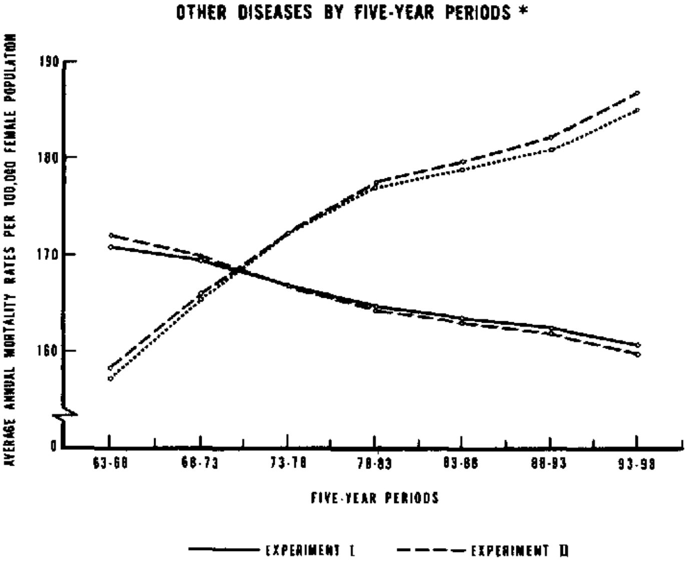
PROJECTIONS OF AVERAGE ANNUAL MORTALITY CAUSED BY CARDIOVASCULAR DISEASES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 22

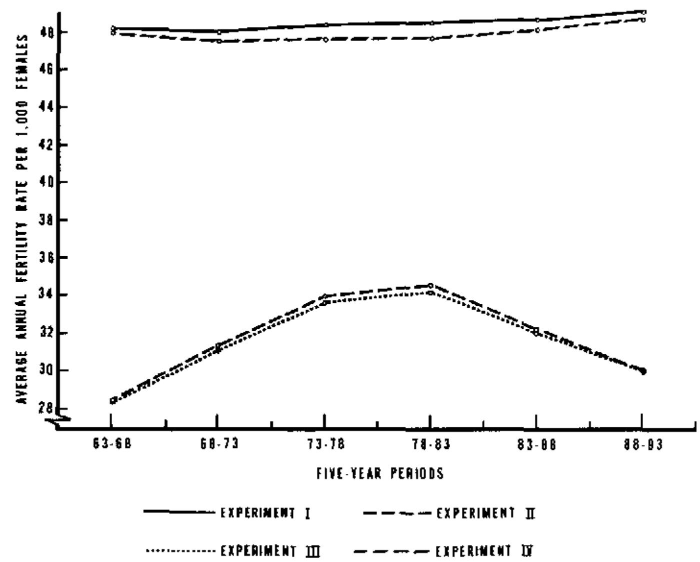
PROJECTIONS OF AVERAGE ANNUAL MORTALITY CAUSED BY ALL OTHER DISEASES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 23

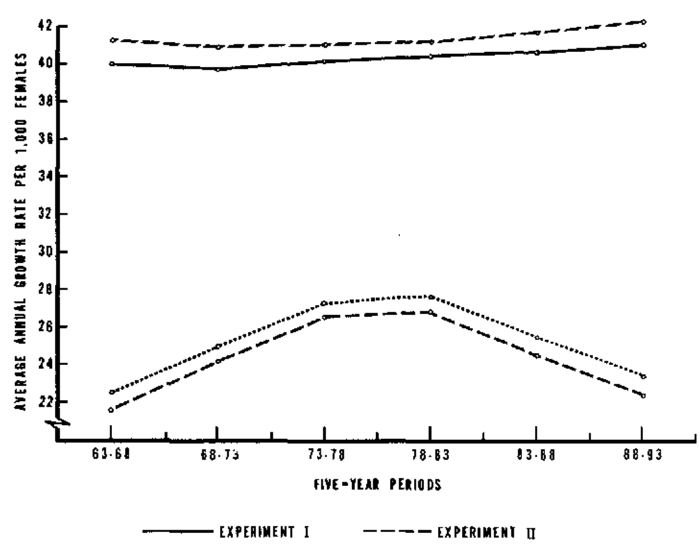
PROJECTIONS OF AVERAGE ANNUAL FERTILITY RATES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 24

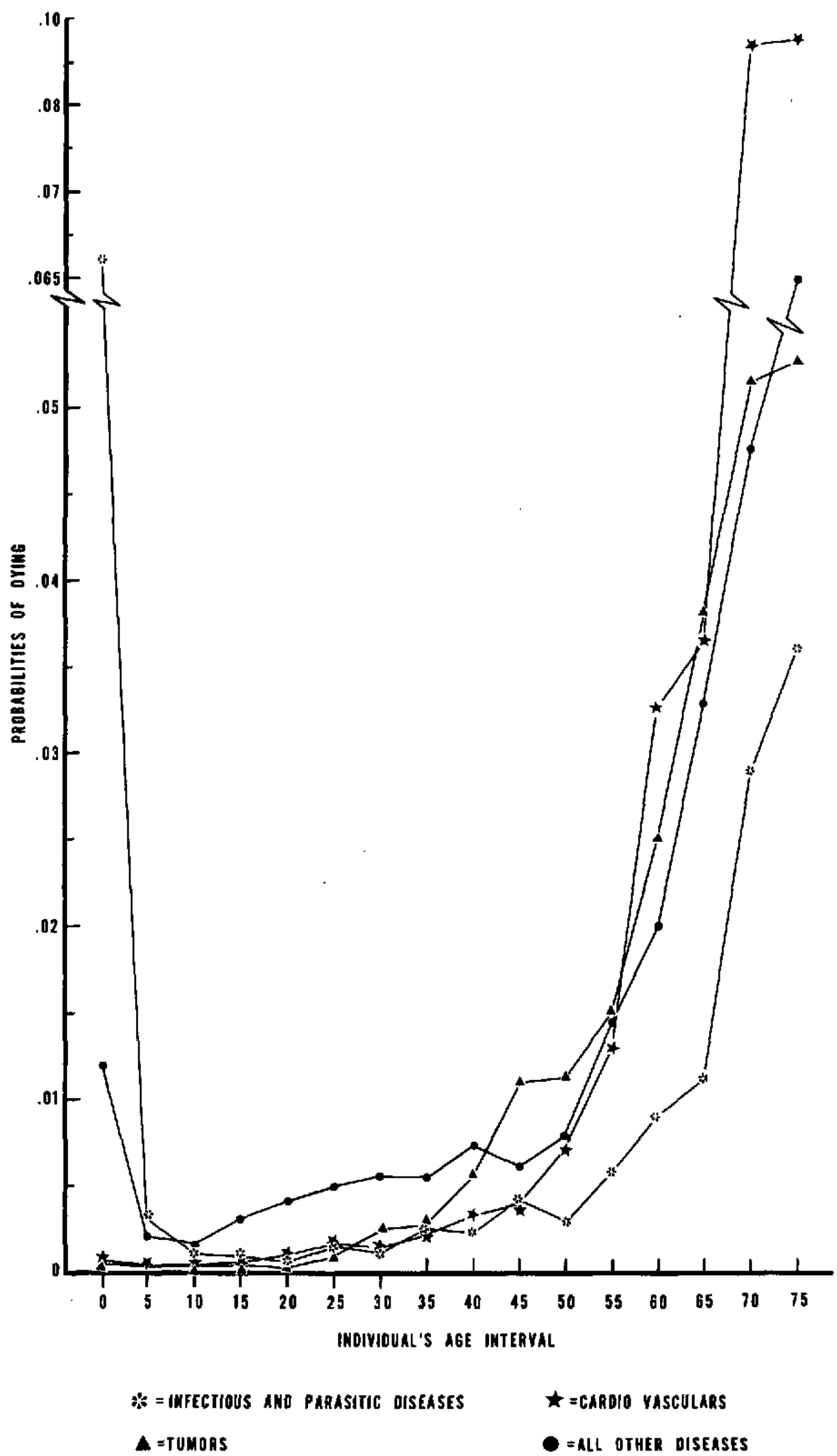
PROJECTIONS OF AVERAGE ANNUAL GROWTH RATES BY FIVE-YEAR PERIODS *



*FEMALE POPULATION OF COSTA RICA

FIGURE 25

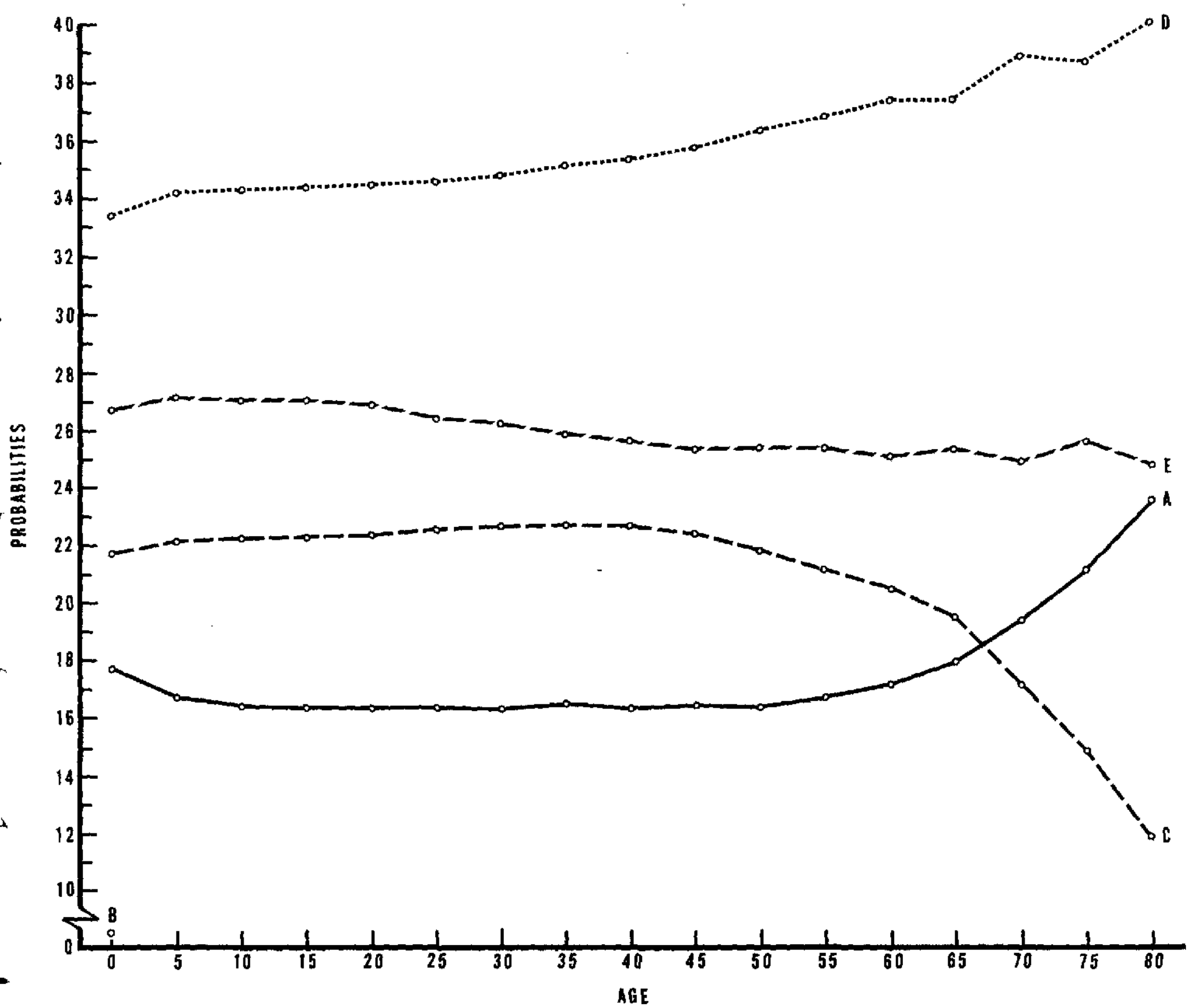
PROBABILITIES AT A GIVEN AGE OF DYING FROM A SPECIFIC GROUP OF DISEASES*



*BASED ON 1963 AGE-SPECIFIC MORTALITY RATES FOR THE FEMALE POPULATION OF COSTA RICA

FIGURE 26

PROBABILITIES, BY AGE, OF EVENTUAL DEATH FROM SPECIFIC GROUPS OF CAUSES*



(A)=INFECTIOUS AND PARASITARY DISEASES
(B)=DISEASES OF EARLY INFANCY
(C)=TUMORS
(D)=CARDIOVASCULAR DISEASES
(E)=ALL OTHER DISEASES

*BASED ON 1963 AGE-SPECIFIC MORTALITY RATES FOR THE FEMALE POPULATION OF COSTA RICA

Discussion: Results in the Steady State

The steady state situation has been defined to exist from that point of time at which the percentage distribution of the population has become virtually constant.

Tables 4 and 5 contain the steady state percentage composition of death and of the death rate per 100,000 by groups of causes of death, respectively, for the four experimental situations in steady state. Thus, the long-range criteria outcomes for the percentage composition of death and of the death rate may be compared for the four hypothetical program options associated with the four experiments.

Table 4, for example, shows that the baseline (Experiment 1) percentage of all deaths due to infectious and parasitic diseases is 48.3; however, if the outlined alteration of the specific mortality in the first age group were to be made with no change in SFRA's (Experiment 4), the new percentage of all deaths due to infectious and parasitic diseases could be 34.5. Thus, the corresponding program change would effect a decrease of 13.8 in the percentage of deaths due to these causes. (On the other hand, the percentage of deaths due to other causes would go up, of course, since the total percentage is always 100.)

If the SMRAG's are all fixed but the SFRA's vary, it is seen that the redistribution of the population by age ultimately (in steady state) causes the percentage of deaths due to cancer to rise from 8.5 to 16.1 per cent because of the older population resulting from decreased SFRA's.

Turning to Table 5, again examining the case of fixed SFRA's and reduced versus baseline SMRAG's (Experiment 4 outcome versus Experiment 1 outcome), it is seen that the input change within age interval 1 of 1,450 deaths per 100,000 population due to diseases in Group A to 695 deaths per 100,000 due to these same causes has effected, in steady state, a specific mortality rate reduction for Group A diseases from 373.1 per 100,000 to 206.7 per 100,000 and, concomitantly, has caused slightly decreased specific mortality rates within the other disease groups as well (except Group B, or diseases of early infancy). The impact on the total mortality rate has been a decrease of 172.3 per 100,000.

On the other hand, if the SMRAG's are constant but the SFRA's are changed, it is seen that ultimately the decrease in specific fertility results in a death rate <u>decrease</u> of 71.4 per 100,000 in Group A and 40.6 per 100,000 in Group B, but an <u>increase</u> in death rate of 88.1 for Group C, 131.2 for Group D, and 74.7 for Group E. Obviously, infectious and parasitic diseases and illnesses of early infancy will not take the same toll in an older population; the other classes --tumors, cardiovascular diseases, and other causes-- will be more prevalent.

Table 6 gives the steady state total death rates, total fertility rates, and growth for the four experimental situations.

Table 4

Experimental Design Outcome: A Steady State Percentage Distribution of Death by Group of Diseases

SMRA		I. SFRA--baseline					II. SFRA--reduced					Difference
		A	B	C	D	E	A	B	C	D	E	
I. SMRAG--Baseline 1963, fixed	A	48.3					31.7					- 16.6
	B		13.2					6.4				- 6.8
	C			8.5					16.1			+ 7.6
	D				10.2					22.0		+ 11.8
	E					19.8					23.8	+ 4.0
II. SMRAG--1963, reduced	A	34.5					24.0					- 10.5
	B		17.5					7.8				- 9.7
	C			10.4					12.7			+ 7.3
	D				12.4					23.8		+ 11.4
	E					25.2					26.7	+ 1.5
Difference		13.8	+4.37	+1.0	+2.2	+5.4	-7.7	+1.4	+1.6	+1.8	+2.9	

Table 5

EXPERIMENTAL DESIGN OUTCOMES: STEADY STATE DEATH RATES/100,000 FOR BASELINE COSTA RICA DATA, 1963, AND THREE SIMULATED PROGRAMS

		I. SFRA--baseline						II. SFRA--reduced						Death rate alteration
		A	B	C	D	E	Total	A	B	C	D	E	Total	
I. S M R A G -- Baseline	A	373.1						301.7						-71.4
	B		102.1						61.5					-40.6
	C			65.4						153.5				+88.1
	D				78.4						209.6			+131.2
	E					153						227.7		+ 74.7
	Total						772						954	+182
II. S M R A G -- Reduced	A	206.7						198						-8.7
	B		105.3						64					-41.3
	C			62.2						146				+83.8
	D				74.1						197			+122.9
	E					151.4						220		+68.6
	Total						599.7						825	+225.5
Death rate alteration		-166.4	+3.3	-3.2	-4.3	-1.7	-172.3	-103.7	+2.5	-7.5	-12.6	-7.7	-129	

Table 6-A

Experimental Design Steady State Total Death Rate Outcome

		I. SFRA--baseline	II. SFRA--reduced	Difference
SMRAG	I	7.72	9.54	+1.82
	II	6.00	8.25	+2.25
	Difference	-1.72	-1.29	

Table 6-B

Experimental Design Steady State Total Fertility Rate Outcome

		I. SFRA--baseline	II. SFRA--reduced	Difference
SMRAG	I	49.44	29.21	-20.23
	II	49.30	29.31	-19.99
	Difference	-.14	+.10	

Table 6-C

Experimental Design Steady State Growth Rate Outcome

		I. SFRA--baseline	II. SFRA--reduced	Difference
SMRAG	I	41.72	19.67	-22.05
	II	43.30	21.06	-22.24
	Difference	+1.58	+13.90	

Conclusion

A Markovian model of the birth-life-death process has been developed that relates the input decision variables of specific mortality rates by age and group of diseases (SMRAG's) and specific fertility rates by age (SFRA's) to output criteria involving life expectancy, the time-dependent structure of mortality by age and cause of death, and other time-dependent demographic structures.

The concept of experimentation in the form of computer simulation to determine the impact of health services programs has been introduced. If one knows the cost of a program, and its likely alteration of the SMRAG's and SFRA's, the simulation will determine effectiveness as measured in terms of the output criteria.

The distinction between transient and steady state solution (and situations) has been made. The immediate effects of the application of a program are demonstrable through examination of the transient values of the criteria. Long-range comparisons among programs may be made by examining the steady state values of the output criteria effected by the various programs.

The present methodology may be used in the following ways:

- To make a dynamic analysis of mortality structures by age intervals and disease groupings under various hypothesis of SMRAG's and SFRA's;
- To simulate the behavior of population structures and other vital criteria under various experimental (program) conditions;
- As a simple computational tool, to estimate the gain in life expectancy effected by different programs of specific mortality reductions;
- In conjunction with available computer programs at the Johns Hopkins University, to estimate the manpower and general resource requirements necessitated by various programs;
- To serve as an educational tool for students in Public Health Services.

There are two analytic innovations in the development of this model that make it mathematically suited for the range of studies mentioned in the above list: (1) the ability to calculate the required transition probabilities in the Markov matrix C in terms of SMRAG's and SFRA's through formulas adapted from demographic articles and writings (1, 6, 8); and (2) the use of the "absorbing barriers" representation of Markov chains so that deaths due to an arbitrary number of causes (in this case the five groups of diseases) may be tabulated explicitly, and thus the effects of all the SMRAG's and SFRA's on any defined disease population (i.e., cause of death grouping) are determined.

APPENDIX I

ESTIMATION OF THE ABSORBING AND SURVIVAL PROBABILITIES

Definitions

M_{ij} = Annual specific mortality rate by age interval i (i=1, 2,..., 18) and by group of causes of death j (j= A, B, C,..., E)

$M_{i.}$ = $\sum_j M_{ij}$ (Annual specific mortality by age interval)

n_i = Size of age interval i, in this case, $n_i = n$ for i=1, 2,..., 17

A_i = $\sum_{j=1}^{i-1} n_j$ (Size of the first i-1 age intervals)

${}_nq_{i,j}$ = Probability that an individual of exact age A_i at the beginning of the age interval i will die before reaching the end of that interval by any of the diseases of Group j

${}_nq_{i,.}$ = $\sum_j {}_nq_{i,j}$

$P_{i,i+1}$ = $(1- \quad q_{i,.})$ = (Probability of an individual of exact age A_i at the beginning of the i^{th} age interval surviving to the end of that interval and entering the next age interval i + 1)

$S_{i,i\ 1}$ = Probability of an individual whose age is contained in the age interval i surviving to the end of that interval and entering the next age interval i 1

${}_nq^*_{i,.}$ = $(1 - S_{i,i\ 1})$ = (Probability of an individual whose age is contained in the age interval i dying before reaching the end of that interval)

${}_nq^*_{i,j}$ = Probability of an individual whose age is contained in the age interval i dying before reaching the end of that interval by any of the diseases of Group j

Definitions (Cont'd)

$_n{_i}a_i$ = Average number of years lived in the i^{th} interval by an individual who dies within it

$_nL_i$ = The expression from the life table that means the number of person-years lived during the age interval i by the cohort of births assumed. (i = 1, 2,..., w)

l_i = Number of persons living at the beginning of the age interval i, out of a total cohort (l_1) of births that is assumed

An estimate of $_nq_{i,.}$ and $_nq_{i,j}$ can be made by the following function given by Chiang (2) :

$$_nq_{i,.} = \frac{n_iM_i}{1+(1-a_i)\ n_iM_i} \qquad i=(1,\ 2,\ \ldots,\ 17)$$

$$_nq_{i,j} = \frac{n_iM_{i,j}}{1+(1-a_i)\ n_iM_i} \qquad \begin{array}{l} i= (1,\ 2,\ldots,\ 17) \\ j= (A,\ B,\ C,\ D,\ E) \end{array}$$

where the value of a_i estimated from the expression

$$a_i = \frac{_nL_i - n_i l_{i+1}}{n_id_i}$$

$$d_i = l_i - l_{i+1}$$

so that the above probabilities of dying are referred to individuals of exact age A_i at the beginning of the age interval that will be used in the estimation of life expectancy. For the purpose of simulating experiments by the Markov model, however, what is needed are the probabilities of dying for individuals of ages contained in the interval. This can be obtained by applying the following expression:

$$S_{i,i+1} = \frac{{}_nL_{i+1}}{{}_nL_i} \qquad i= (1,2,\ldots,17)$$

which, in probabilistic terms for the use of the present model, can be writen as

$$S_{i,i+1} = P_{i,i+1} \frac{(P_{i+1,i+2} + a_{i+1}\, q_{i+1,.})}{(P_{i,i+1} + a_i\, q_{i,.})} \qquad i=(1,2,\ldots,16)$$

$$S_{i,i+1} = \frac{P_{i,i+1}\,(n_{i+1}\, a_{i+1})}{n_i\,(P_{i,i+1} + a_i\, q_{i,.})} \qquad i=17$$

$$S_{i,i+1} = 0 \qquad i=18$$

The estimate of ${}_nq^*_{i,.}$ is given by

$${}_nq^*_{i,.} = (1 - S_{i,i+1}) \qquad i=(1,2,\ldots 18)$$

and the estimation of its components by

$${}_nq^*_{i,j} = {}_nq^*_{i.} \left(\frac{q_{ij}}{\sum_j q_{ij}} \right)$$

APPENDIX II

ESTIMATION OF THE NUMBER OF BIRTHS

Definitions

$W_{i,t}$ = Female population in the age interval i at time t

F_i = Annual specific fertility rate for age interval i ($F_i \neq 0$, i=3,9)

f = Female fraction of the newborn

u = Real time measured from zero at the start of a simulation

n = Time interval (5 years in the present case)

U_t = Value of u at the beginning of the t^{th} time period ($U_t = (t-1)n$)

${}_5L_0$ and ℓ_1 are defined in Appendix I

The total number of newborns in the period $u_{t+1} - u_t = 5$
is given by

$$\sum_{i=3}^{10} \frac{(W_{i,t} + S_{i-1,t} W_{i-1,t})}{2} \quad 5\, F_i$$

and since the probability of a newborn surviving his age interval is $\frac{{}_5L_1}{5\ell_1}$

and the female fraction of the newborn is f, then the total number of newborns that will survive the period is

$$\sum_{i=3}^{10} \left(\frac{W_{i,t} + W_{(i-1),t}\, S_{i-1,i}}{2} \right) F_i \quad 5\, \frac{{}_5L}{5\ell} \quad f$$

which can be written as

$$\sum_{i=2}^{10} W_{i,t} \left(F_i + S_{i,i+1} F_{i+1}\right)\left(\frac{5}{2} \quad P_1^* f\right) \quad i=2,3,\ldots,10$$

noting that

$$F_i = 0 \text{ for } i=2$$

$$F_{i+1} = 0 \text{ for } i=10$$

where

$$P_1^* = (P_{1,2} + q_{1,.}a_1)$$

and

$$S_{i,i+1} = P_{i,i+1} \left(\frac{P_{i+1,i+2} + a_{i+1} \ q_{i+1,.}}{P_{i,i+1} + a_i \ q_{i,.}} \right)$$

The term $b_{i1} = \frac{5}{2} \quad P_1^*\left(f \quad F_i + S_{i,i+1} F_{i+1}\right)$ for i=3,4,...,10, is the one that appears in the i^{th} row of the column of b's in the transition matrix given in table 1-A.

APPENDIX III

ESTIMATION OF LIFE EXPECTANCY

The life expectancy e_i of a person exactly A_i years old is classically estimated through the use of life table functions, expressed by the formula

$$e_i = \frac{T_i}{\ell_i}$$

ℓ_1 is the number of people in a hypothetical cohort. (Usually ℓ_1 = 100,000.)

ℓ_i is the expected number of people within the cohort who will live to be A_i years or older.

T_i is the expected total number of years lived from age A_i on, by the ℓ_i people.

A general expression for e_i is:

$$e_i = \sum_{k=i}^{\omega-1} n_k P_k^* \prod_{r=0}^{k-1} p_{r,r+1} + \prod_{r=0}^{\omega-1} p_{r,r+1}/M_\omega \qquad , i = 1,2,\ldots,\omega-1$$

$$e_i = 1/M_\omega \qquad , i = \omega$$

where

$P_i^* = p_{i,i+1} + a_i q_{i,\cdot}$

n_i = Size of the i^{th} age interval

$P_{i,i+1}$ = Estimated probability of a person surviving to age $A_i + n_i$ (given, of course, that he has survived to his A_i^{th} birthday)

$$A_i = \sum_{j=o}^{i-1} n_j$$

$$q_{i,.} = (1 - P_{i,i+1}), \qquad i=1, \quad W - 1$$

a_i = The average fraction of the amount of years lived by those who reach age A_i but die before age $A_i + n_i$

M_w = The crude mortality rate in the last age interval

Note that

$$P_{i,i+1} = 1 - \sum_{j=A}^{E} q_{i,j}$$

where

$q_{i,j}$ = The probability of a person in age bracket i dying of group j

and the values for $q_{i,j}$ have been estimated by expression given by Chiang

$$q_{i,j} = \frac{M_{i,j} \; n_i}{1 + (1-a_i) \; n_i \; M_{i.}} \qquad \begin{array}{l} j= \; A, \; E \\ i= \; 1,w-1 \end{array}$$

where

$$M_{i.} = \sum_{j=A}^{E} M_{ij}$$

M_{ij} = specific mortality rate due to group j in age bracket i

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